Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2015

Mathematics MPC1

Unit Pure Core 1

Wednesday 13 May 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You must **not** use a calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The use of calculators is **not** permitted.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The line AB has equation $3x + 5y = 7$.

(a) Find the gradient of AB.

[2 marks]

(b) Find an equation of the line that is perpendicular to the line AB and which passes through the point $(-2, -3)$. Express your answer in the form $px + qy + r = 0$, where p, q and r are integers.

[3 marks]

(c) The line AC has equation $2x - 3y = 30$. Find the coordinates of A.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE

Answer space for question 1

2 The point P has coordinates $(\sqrt{3},\ 2\sqrt{3})$ and the point Q has coordinates $(\sqrt{5},\ 4\sqrt{5})$. Show that the gradient of PQ can be expressed as $n + \sqrt{15}$, stating the value of the integer n.

[5 marks]

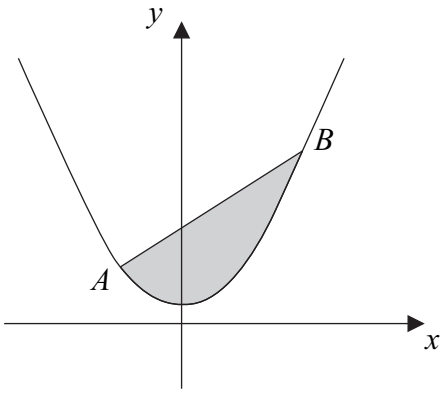
QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

3 The diagram shows a sketch of a curve and a line.



The curve has equation $y = x^4 + 3x^2 + 2$. The points $A(-1, 6)$ and $B(2, 30)$ lie on the curve.

(a) Find an equation of the tangent to the curve at the point A.

[4 marks]

(b) (i) Find $\int_{-1}^{2} (x^4 + 3x^2 + 2)\, \mathrm{d}x$.

[5 marks]

(ii) Calculate the area of the shaded region bounded by the curve and the line AB.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 A circle with centre C has equation $x^2 + y^2 + 2x - 6y - 40 = 0$.

(a) Express this equation in the form

$$(x - a)^2 + (y - b)^2 = d$$

[3 marks]

(b) (i) State the coordinates of C.

[1 mark]

(ii) Find the radius of the circle, giving your answer in the form $n\sqrt{2}$.

[2 marks]

(c) The point P with coordinates $(4, k)$ lies on the circle. Find the possible values of k.

[3 marks]

(d) The points Q and R also lie on the circle, and the length of the chord QR is 2. Calculate the shortest distance from C to the chord QR.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

5 (a) Express $x^2 + 3x + 2$ in the form $(x + p)^2 + q$, where p and q are rational numbers.
[2 marks]

(b) A curve has equation $y = x^2 + 3x + 2$.

(i) Use the result from part **(a)** to write down the coordinates of the vertex of the curve.
[2 marks]

(ii) State the equation of the line of symmetry of the curve.
[1 mark]

(c) The curve with equation $y = x^2 + 3x + 2$ is translated by the vector $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

Find the equation of the resulting curve in the form $y = x^2 + bx + c$.
[3 marks]

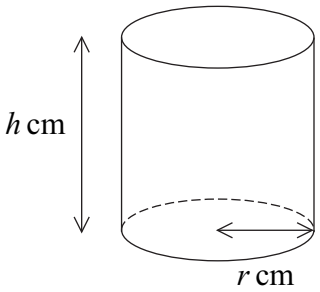
QUESTION PART REFERENCE

Answer space for question 5

QUESTION PART REFERENCE

Answer space for question 5

6 The diagram shows a cylindrical container of radius r cm and height h cm. The container has an **open** top and a circular base.



The **external** surface area of the container's curved surface and base is 48π cm^2.

When the radius of the base is r cm, the volume of the container is V cm^3.

(a) (i) Find an expression for h in terms of r.

[3 marks]

(ii) Show that $V = 24\pi r - \frac{\pi}{2}r^3$.

[2 marks]

(b) (i) Find $\frac{\mathrm{d}V}{\mathrm{d}r}$.

[2 marks]

(ii) Find the positive value of r for which V is stationary, and determine whether this stationary value is a maximum value or a minimum value.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

7 (a) Sketch the curve with equation $y = x^2(x - 3)$.

[3 marks]

(b) The polynomial $\mathrm{p}(x)$ is given by $\mathrm{p}(x) = x^2(x - 3) + 20$.

(i) Find the remainder when $\mathrm{p}(x)$ is divided by $x - 4$.

[2 marks]

(ii) Use the Factor Theorem to show that $x + 2$ is a factor of $\mathrm{p}(x)$.

[2 marks]

(iii) Express $\mathrm{p}(x)$ in the form $(x + 2)(x^2 + bx + c)$, where b and c are integers.

[2 marks]

(iv) Hence show that the equation $\mathrm{p}(x) = 0$ has exactly one real root and state its value.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 7

QUESTION PART REFERENCE

Answer space for question 7

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Answer space for question 7

8 A curve has equation $y = x^2 + (3k - 4)x + 13$ and a line has equation $y = 2x + k$, where k is a constant.

(a) Show that the x-coordinate of any point of intersection of the line and curve satisfies the equation

$$x^2 + 3(k - 2)x + 13 - k = 0$$

[1 mark]

(b) Given that the line and the curve do not intersect:

(i) show that $9k^2 - 32k - 16 < 0$;

[3 marks]

(ii) find the possible values of k.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE

Answer space for question 8

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END OF QUESTIONS